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Rayat Shikshan Sanstha's

Sadguru Gadge Maharaj College, Karad

(An Empowered Autonomous College)

12141

[redacted]

SGMCK

(AN EMPOWERED AUTONOMOUS COLLEGE)

Information to be filled by Student
(विद्यार्थ्याने भरावयाचा रकाना)

Day and Date : Monday 03/11/2025

Language of Answer : English.

Examination : M.Sc-II-Sem-II OCT/NOV 25

Question Paper Code No. : MJ-MMT23-302

Subject : Algebra

Paper No. MJ-MMT23-302

Section : –

(Office

Q. No.	Examiner Marks	Moderator Marks
1	14	
2	09	
3	16	
4	16	
5	16	
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7	13	
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Total	75	
Signature		

Q. C. - O. K.

IMPORTANT

As per Maharashtra Act No. XXXI of 1982 (7 & 8) whoever is found in or near an examination hall by the invigilator or any her person appointed to supervise the conduct of the examination, copying answers to the question paper set at the xamination, from any book, notes or answer papers of other candidates, of appearing at the examination for any other andidates or using any other unfair means, shall, on conviction, be punished with imprisonment for a term which may xtend to six months, or with fine which may extend to five hundred rupees or with both.

Whoever abets any offence punishable under this Act shall be punishable with the punishment provided for the offence.

चना : विद्यापीठ नियमानुसार जो कोणी परिक्षा हॉलजवळ किंवा हॉलमध्ये पर्यवेक्षकांना किंवा परीक्षेसाठी नेमलेल्या कर्मचाऱ्यांना उत्तरांची पुस्तक ोट्स किंवा अन्य विद्यार्थ्यांच्या उत्तरपत्रिकेतून नक्कल करतांना आढळून येईल किंवा जवळ परीक्षा गैरव्यवहारासाठी वापरता येईल असे अक्षेपार्ह ाहित्य बाळगत असेल तर सदरची बाब परीक्षा प्रमाद मानून संबंधित व्यक्ति शिक्षेस पात्र राहील.

महत्वाच्या सूचना

१. उत्तरपत्रिकेमध्ये अन्यत्र कोठेही बैठक क्रमांक (दिलेल्या जागेखेरीज) लिहू नये, तसेच अन्यत्र कोणत्याही प्रकारच्या खुणा करू न
अशा खुणा आढळल्यास तो ओळख पटविण्याचा प्रयत्न मानन्यात येईल व असा परीक्षा प्रमाद शिक्षेस पात्र राहील.
२. उत्तरास प्रश्न क्रमांक व उपप्रश्न क्रमांक लिहणे अनिवार्य आहे.
३. पूर्ण प्रश्नांचे उत्तर सलग व सुवाच्च अक्षरात लिहावे.
४. अवाचनीय अक्षरातील उत्तर शुन्य गुणासाठी ग्राह्य मानण्यात येईल.
५. आकृत्याखेरीज सर्व उत्तरासाठी फक्त निळ्या शाईचा वापर करावा.

IMPORTANT

1. Do not write seat number anywhere except at the space provided for it. If seat number or any marks, sym which indicates identity is/are found anywhere, it will be treated as malpractice and lapse. Such malpractic lapse will be punishable under Maharashtra University Act 1994.
2. It is necessary to write the Question No. and sub Question Number before answer.
3. Write the complete answer in sequential manner and then start the next question in readable handwriting.
4. Answer in an illegible & undecipherable handwriting are liable to be marked as zero.
5. Use of blue color ink (expect diagrams) is permitted.

Q. No.	Verificaton Marks	Revaluation M
1		
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Q. No.	1(a)	B)		TOTAL				TOTAL
Marks	06	08	=	14				

Q. No. 1.

A]

Q.1. 1] B] 30

2] A] G

3] A) 7

4] A] x^2+1.

6

Q.1] B]

1] True

2] True.

3] False.

4] True.

08

Q. No.	3 a)	b)		TOTAL				TOTAL
Marks	08	08	=	16				

Q. No 2]

Q.3] a] **Proof:-** To prove that,

Two subnormal series of group G have isomorphic refinement.

Consider,

Two subnormal series.

$\{e\} = H_0 < H_1 < H_2 < H_3 \ldots\ldots < H_n = \{G\}$

and

$\{e\} = K_0 < K_1 < K_2 < K_3 \ldots\ldots < K_m = \{G\}$.

Then we know that..

$H_i \trianglelefteq H_{i+1}$ and $K_j \trianglelefteq K_{j+1}$ for each $1 \le i \le n$, $1 \le j \le m$

Then,

$H_i(H_{i+1} \cap K_j)$ and $H_i(H_{i+1} \cap K_{j+1})$ for $1 \le i \le n$ & $1 \le j \le m$

we get,

~~$H_{0,0}$~~

$H_{i,j} = H_{0,0} \trianglelefteq H_{0,1} \trianglelefteq H_{0,2} \trianglelefteq \ldots\ldots \trianglelefteq H_{0,m} = H_{1,0} = H_1$

$= \{e\}$ $\quad H_{1,1} \trianglelefteq H_{1,2} \trianglelefteq \ldots\ldots \trianglelefteq H_{1,m} = H_{2,0} = H_2$

$\vdots$

$H_{n,1} \trianglelefteq H_{n,2} \trianglelefteq \ldots\ldots \trianglelefteq H_{n,m} = H_n = \{G\}$ ----(i)

Then the factor of the group are written as.

$$\left\{ \frac{H_{i+1}}{H_i} \Big/ \begin{matrix} 1 \le i \le n-1 \\ 1 \le j \le m \end{matrix} \right\} \rightarrow \left\{ H_i(H_{i+1} \cap K_j)' \Big/ \begin{matrix} 1 \le i \le n-1 \\ 1 \le j \le m \end{matrix} \right\} \text{----(i)}$$

Now,

consider for,

$K_j \trianglelefteq K_{j+1}$. for each $1 \le j \le m$

Q. No.				TOTAL				TOTAL
Marks								

. No.

Then,

$k_j(H_i \cap k_{j+1})$ and $k_j(H_{i+1} \cap k_{j+1})$.

Then,

we get,

$k_{j,j} = k_{0,0} \trianglelefteq k_{0,1} \trianglelefteq k_{0,2} \trianglelefteq \cdots \trianglelefteq H_{0,n} = K_{1,0} = K_1$

$= \{e\}$ $\quad k_{1,1} \trianglelefteq k_{1,2} \trianglelefteq \cdots \trianglelefteq k_{1,n} = K_{2,0} = K_2$

$\vdots$

$k_{m,1} \trianglelefteq k_{m,2} \trianglelefteq \cdots \trianglelefteq k_{m,n} = K_m = \{G\}$ ---(iii)

Then the factor group for the above seris is given as :-

$$\left\{ \frac{k_{j+1}}{k_j} \middle/ \begin{matrix} 1 \le i \le \cancel{m-1}\, n \\ 1 \le j \le m-1 \end{matrix} \right\}$$

$$\Rightarrow \left\{ k_j(H_i \cap k_{j+1}) \middle/ \begin{matrix} 1 \le i \le n \\ 1 \le j \le m-1 \end{matrix} \right\} \text{ ---(iv).}$$

observing eq^{n}s (ii) & (iv) we ~~al~~ see that.
This series are isomorphic refinements of each other.
Now on deleting repeated series from eq^n (ii) & (iii)
we get.
Two subnormal series of group G have isomorphic refinement.

08 Which completes the proof.

Q. No.				TOTAL				TOTAL
Marks								

Q. No.

Q. 3. b]. Sol^n:- We have to find the no. of elements of all possible orders in symmetric group S_4.

Firstly we will observe the partitions of S_4.

$S_4 = 4 \quad = \cancel{lcm} (4) = 4$

$3+1 \quad = lcm(3,1) = 3$

$2+2 \quad = lcm(2,2) = 2$

$2+1+1 \quad = lcm(2,1,1) = 2$

$1+1+1+1 \quad = lcm(1,1,1,1) = 1$

Then,

Partitions of S_4 are :- 1, 2, 2, 3, & 4.

Now,

To calculate no. of elements of order 1 is S_4:

$$\Rightarrow \frac{4!}{(1)^4 \times (4)!} = \frac{1 \times 2 \times 3 \times 4}{1 \times 1 \times 2 \times 3 \times 4} = 1$$

Now,

To calculate no. of elements of order 2 in S_4:

Now we see that there are 2 partitions for 2

firstly.

To calculate no. of elements of order (2,1,1) in S_4:-

$$\Rightarrow \frac{4!}{1^{(2)} \times 2^{(1)} \times (2)! \times (1)!}$$

$$\Rightarrow \frac{1 \times 2 \times 3 \times \cancel{4}}{1 \times \cancel{2} \times \cancel{2} \times 1}$$

$$\Rightarrow 6.$$

Now

To calculate no. of elements of order (2,2) in S_4:-

$$\Rightarrow \frac{4!}{(2)^2 \times (2)!}$$

$$\Rightarrow \frac{1 \times \cancel{2} \times 3 \times \cancel{4}}{\cancel{4} \times 1 \times \cancel{2}} = 3$$

Q. No.				TOTAL				TOTAL
Marks								

Q. No.

Now,

To calculate no. of elements of order 3 in S_4:

$$\Rightarrow \frac{4!}{1^{(1)} \times 3^{(1)} \times (1)! \times (1)!}$$

$$\Rightarrow \frac{1 \times 2 \times \cancel{3} \times 4}{1 \times \cancel{3} \times 1 \times 1}$$

$$\Rightarrow \underline{\underline{8}}$$

Now,

To calculat no. of element of order 4 in S_4:

$$\Rightarrow \frac{4!}{4^{(1)} \times (1)!}$$

$$\Rightarrow \frac{1 \times 2 \times 3 \times \cancel{4}}{\cancel{4} \times 1}$$

$$\Rightarrow \underline{\underline{6}}$$

Therefore we see that,

no. of element of order 1 in S_4 = 1

no. of elements of order 2 in S_4 = 6 + 3 = 9

no. of elements of order 3 in S_4 = 8

no. of elements of order 4 in S_4 = 6

Adding up this no. of element we get $o(S_4)$.

$o(S_4) = 4! = 1 \times 2 \times 3 \times 4 = 24$

$o(S_4) = 24$.

and,

$1 + 9 + 8 + 6 = \cancel{14}\ \underline{\underline{24}}$

Which is required answer.

08

Q. No.	4 a)	b)		TOTAL				TOTAL
Marks	08	08	=	16				

Q. No. 4.

4. a) State & prove Butterfly Lemma :-

statement :- Let, H and k be the subgroups of group G. and H^* & k^* be the normal subgroups of H and k respectively then :

(i) $H^*(H \cap k^*) \trianglelefteq H^*(H \cap k)$

(ii) $k^*(H^* \cap k) \trianglelefteq k^*(H \cap k)$

(iii) $\dfrac{H^*(H \cap k)}{H^*(H \cap k^*)} \cong \dfrac{k^*(H \cap k)}{k^*(H^* \cap k)} \cong \dfrac{H \cap k}{H^*(H \cap k)\, k^*(H \cap k)}$

→ Proof :- (i) $H \cap k \leq H$, $H^* \trianglelefteq H$.

$H^*(H \cap k) \leq H$

(ii) $H \cap k \leq k$, $k^* \trianglelefteq k$

$H^*(H \cap k) \leq k$

(iii) $H^* \cap k \leq k$ and $H^* \cap k \trianglelefteq H$.

$H^* \cap k \trianglelefteq H \cap k$

~~(iv) $H k^* \cap k$~~ (iv) $H \cap k^* \leq H$ and $H \cap k^* \trianglelefteq k$

~~$H^* \cap$~~ $H \cap k^* \trianglelefteq H \cap k$

we get,

$H^*(H \cap k)\, k^*(H \cap k) \trianglelefteq H \cap k$

i.e. $\dfrac{H \cap k}{H^*(H \cap k)\, k^*(H \cap k)}$ well defined

Now,

consider,

$L = H^*(H \cap k)\, k^*(H \cap k)$.

i.e.

$L \trianglelefteq H \cap k$.

$\Rightarrow$ ~~$\dfrac{H \cap k}{H^*(H \cap k}$~~ $\dfrac{H \cap k}{L}$

Q. No.				TOTAL				TOTAL
Marks								

Now.

Define,

$$\Phi : H^*(H\cap K) \longrightarrow \frac{H\cap K}{H^*(H\cap K)K^*(H\cap K)} \quad \text{i.e. } \frac{H\cap K}{L}.$$

is defined by,

$$\Phi(hx) = L(x) \quad ; h\in H^* \text{ and } x \in H\cap K.$$

Then,

(i) Φ is well defined an one-one :-

Let, h_1x_1 & $h_2x_2 \in H^*(H\cap K)$ such that,

$\Leftrightarrow h_1x_1 = h_2x_2$

$\Leftrightarrow h_1h_2^{-1} = x_2x_1^{-1}$

$\Leftrightarrow h_1h_2^{-1} = x_2x_1^{-1} \in H\cap K$ & $h_1h_2^{-1} \in H^*$

$\Leftrightarrow \cancel{h_1h_2^{-1}} \in \cancel{H}$.

$\Leftrightarrow x_2x_1^{-1} \in H^*\cap(H\cap K) \subset L$.

$\Leftrightarrow x_2x_1^{-1} \in L$.

$\Leftrightarrow L(x_2) = L(x_1)$ ---- $[ab^{-1}\in H \Rightarrow aH = Hb]$.

$\Leftrightarrow \Phi(h_2x_2) = \Phi(h_1x_1)$

This shows that, Φ is well defined & one-one

(ii) Φ is onto :-

Let, $Lx \in \frac{H\cap K}{L}$

then Let, $x \in H\cap K$

and $e \in H^*$

This implies that, $ex \in H^*(H\cap K)$

This shows that,

Φ is onto.

Q. No.				TOTAL				TOTAL
Marks								

Q. No.

(iii) Φ is homomorphism :-

Let, $h_1x_1, h_2x_2 \in H^*(H \cap k)$

Then,

$\Phi(h_1x_1h_2x_2) = \Phi(h_1(x_1h_2)x_2)$

$= \Phi(h_1(h_2x_1)x_2)$

$= \Phi(h_1(h_3x_1)x_2)$ —— [say H_3 for any]

$= \Phi(h_1h_3x_1x_2)$

$= L(x_1).L(x_2)$

$= \Phi(h_1x_1).\Phi(h_2x_2)$

$\therefore$ for, $\Phi(h_1x_1h_2x_2) = \Phi(h_1x_1).\Phi(h_2x_2)$,

$\therefore$ Φ is homomorphism.

Hence, from (i), (ii) & (iii) we see that Φ is onto homorphism

i.e $H^*(H \cap k)$ is homomorphic to $\frac{H \cap k}{L}$;

Now,

$\Phi: \frac{H^*(H \cap k)}{\ker \Phi} \to \frac{H \cap k}{L}$.

them we have to show that,

$\ker \Phi = H^*(H \cap k^*)$

Let,

$\ker \Phi = \{hx / \Phi(x) = L ; hx \in H^*(H \cap k)\}$

$= \{hx / l(x) = L ; hx \in H^*(H \cap k)\}$

$= \{hx / xe^{-1} \in L ; hx \in H^*(H \cap k)\}$

$= \{hx / x \in L ; hx \in H^*(H \cap k)\}$

$= \{hx / hx \in H^*(H \cap k) ; x \in L\}$

$= \{hx / h \in H^*, x \in (H \cap k), x \in L\}$

$= \{hx / h \in H^*, x \in (H \cap k) L\}$

$= \{hx / h \in H^*, \forall x \in (H \cap k^*)\}$

Q. No.				TOTAL				TOTAL
Marks								

$\ker \Phi = \{hx / h \in H^*, x \in (H \cap K^*)\}$
$= \{hx / hx \in H^*(H \cap K^*)\}$

$\rightarrow \ker \Phi = H^*(H \cap K^*)$
This shows that

$$H^*(H \cap K^*) \triangleleft H^*(H \cap K).$$

Now,
Similarlly,
we will prove,
$\ker \Phi = K^*(H^* \cap K)$.
i.e.
$K^*(H^* \cap K)$ is normal in ~~H~~ $K^*(H \cap K)$.
and for $L = H^*(H \cap K) \, K^*(H \cap K)$
we get,

$$\frac{H \cap K}{H^*(H \cap K) \, K^*(H \cap K)} \cong \frac{K^*(H \cap K)}{K^*(H^* \cap K)} \cong \frac{H^*(H \cap K)}{H^*(H \cap K^*)}$$

which is required proof.

Q. No. 4.

Q.4. b] Sol^n:- To derive class eq^n of symmetric group S_3.

We have,

$G = S_3 = \{I, (1\,2), (1\,3), (2\,3), (1\,2\,3), (1\,3\,2)\}$

Then,

firstly we will calculate conjugcy class of S_3.

$S_3 = \{I\} = 1.$

$[cl(I)] = 1.$

Now,

$[cl(1\,2)] = \{I^{-1}(1\,2)I, (1\,2)^{-1}(1\,2)(1\,2), (1\,3)^{-1}(1\,2)(1\,3), (2\,3)^{-1}(1\,2)(2\,3), (1\,2\,3)^{-1}(1\,2)(1\,2\,3), (1\,3\,2)^{-1}(1\,2)(1\,3\,2)\}.$

$= \{I, I, (1\,3)(1\,2)(1\,3), (2\,3)(1\,2)(2\,3), (1\,3\,2)(1\,2)(1\,2\,3), (1\,2\,3)(1\,2)(1\,3\,2)\}$

$= \{I, (1\,2\,3)(1\,3), (1\,3\,2)(2\,3), (2\,3)(1\,2\,3), (1\,3)(1\,3\,2)\}$

$[cl(1\,2)] = \{I, (2\,3), (1\,3), (1\,3), (2\,3)\}$

$\Rightarrow [cl(1\,2)] = cl[(1\,3)] = cl[(2\,3)].$

Now,

$[cl(1\,2\,3)] = \{I^{-1}(1\,2\,3)I^{-1}, (1\,2)^{-1}(1\,2\,3)(1\,2), (1\,3)^{-1}(1\,2\,3)(1\,3), (2\,3)^{-1}(1\,2\,3)(2\,3), (1\,2\,3)^{-1}(1\,2\,3)(1\,2\,3), (1\,3\,2)^{-1}(1\,2\,3)(1\,3\,2)\}$

$= \{I, (1\,2)(1\,2\,3)(1\,2), (1\,3)(1\,2\,3)(1\,3), (2\,3)(1\,2\,3)(2\,3), \cancel{(1\,3\,2)(1\,2\,3)(1\,2\,3)}, I, (1\,2\,3)(1\,2\,3)(1\,3\,2)\}$

Q. No.				TOTAL				TOTAL
Marks								

No.

$= \{I, (2\ 3)(1\ 2), (1\ 2)(1\ 3), (1\ 3)(2\ 3), (1\ 3\ 2)(1\ 3\ 2)\}$

$cl[(1\ 2\ 3)] = \{I, (1\ 3\ 2), (1\ 3\ 2), (1\ 3\ 2), (1\ 2\ 3)\}$

$[cl(1\ 2\ 3)] = [cl(1\ 3\ 2)]$.

Now, derive class eq^n of S_3.

$$\sum_{a \in S_3} cl(a) = S_3$$

$$\sum_{a \in S_3} o[cl(a)] = o(S_3)$$

$$o(cl(I)) + o(cl(1\ 2)) + o(cl(1\ 2\ 3)) = o(S_3)$$

$$1 + 3 + 2 = 3! = 1 \times 2 \times 3$$

$$1 + 3 + 2 = 6$$

8

$$\text{i.e.} \sum_{a \in S_3} o(cl(a)) = o(S_3)$$

Q. No.	5 a)	b)		TOTAL					TOTAL
Marks	08	08	=	16					

Q. No. 5,

5] a]. Proof:- consider, R be the collection of all right cosets of G. $G_1 x$.

$R = \{ \cancel{xg} / \cancel{g \in G} \} . / x \in G \}$.

and now,

Let consider a group action of X on R as

$* : R \times R^X \to R$ is defined by.

$(G_1 * x) y = G_1 (xy)$ then $x \in G$ $y \in R$.

Then,

i) ~~Φ is well defined:-~~

(i) for, $(G_1 * x) e = G_1 (x e)$

$= G_1 x$

(ii) for, $y_1, y_2 \in G$

$(G_1 * x)(y_1 y_2) = G_1 [(x)(y_1 y_2)]$

$= G_1 ((x)(y_1)) * y_2$

$= G_1 x * y_1 * y_2$

$(G_1 * x)(y_1 y_2) = G_1 x * y_1 * y_2$

$\therefore$ X is G-set

As, $|xG|$ is always greather than 1

$|xG| > 1$.

then.

$[G : G_x] / |G|$

$[G : G_x] / p^m$.

p / p^m

and let $p^m = [G : G_x]$.

$p / [G : G_x]$.

08

Q. No.				TOTAL				TOTAL
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Q. No.

Q. No.			TOTAL				TOTAL
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Q. No. 5.

Q. 5. b]. Sol^n:- $G = S_3 = \{I, (1\,2), (1\,3), (2\,3), (1\,2\,3), (1\,3\,2)\}$

$O(S_3) = 3! = 6 = 2 \times 3.$

then,

S_3 has sylow-2 subgroup of order 2 in it

then,

This subgroups are,

$H_1 = \{I, (1\,2)\}$

$H_2 = \{I, (1\,3)\}$

$H_3 = \{I, (2\,3)\}$

Now,

we will show that H_1 is conjugate of H_3.

Let, $x = (1\,3)$ then,

$(1\,3)^{-1} H_1 (1\,3) = (1\,3)^{-1} \{I, (1\,2)\} (1\,3)$

$= (1\,3) \{I, (1\,2)\} (1\,3)$

$= (1\,3) \{I, (1\,2)(1\,3)\}$

$= (1\,3) \{I, (1\,3\,2)\}$

$= \{I, (1\,3)(1\,3\,2)\}$

$= \{I\, (2\,3)\}$

$= H_3.$

$\therefore$ H_1 & H_3 conjugates of each other.

Now for,

H_2 & H_3

Let, $x = (1\,2)$ then,

$(1\,2)^{-1} H_2 (1\,2) = (1\,2)^{-1} \{I, (1\,3)\} (1\,2)$

$= (1\,2) \{I, (1\,3)\} (1\,2)$

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Q. No.

$= (1\,2)\{I, (1\,3)(1\,2)\}$

$= (1\,2)\{I, (1\,2\,3)\}$

$= \{I, (1\,2)(1\,2\,3)\}$

$= \{I, (2\,3)\}$

$(1\,2)^{-1} H_2 (1\,2) = H_3$

$\therefore$ H_2 & H_3 are conjugates of each other.

Now for,

H_1 & H_2

let, $X = (2\,3)$.

$(2\,3)^{-1}(H_1)(2\,3) = (2\,3)^{-1}\{I, (1\,2)\}.(2\,3)$

$= (2\,3)\{I, (1\,2)\}(2\,3)$.

$= (2\,3)\{I, (1\,2)(2\,3)\}$

$= (2\,3)\{I, (1\,2\,3)\}$

$= \{I, (2\,3)(1\,2\,3)\}$

$= \{I, (1\,3)\}$.

$= H_2$.

$(2\,3)^{-1}(H_1)(2\,3) = H_2$

$\therefore$ H_1 & H_2 are conjugates of each other which is required answer

8

Q. No.	7 a)	b)		TOTAL				TOTAL
Marks	08	05	=	13				

Q. No. 7

Q.7 a] <u>Proof</u>:- Let M & N be R-modules $\Phi(x)=0$

and, $0 \in \ker \Phi$

then $\ker \Phi \neq \phi$.

Now,

let, $\Phi(x)=0$ and $\Phi(y)=0$. and $r \in R$.

then,

$$\Phi(rx+y) = \Phi(rx+y)$$
$$= \cancel{\Phi}\ \Phi(rx) + \Phi(y)$$
$$= r\Phi(x) + \Phi(y)$$
$$= r.0 + 0$$
$$= 0$$

$\Phi(rx+y) = 0$.

This proves that $\ker \Phi$ is submodule of M.

Now, $\Phi : M \to \operatorname{Img} \Phi$ is defined by.

$f : k$

$$f(x+k) = \Phi(x) \quad ; x \in M.$$

Then,

(i) <u>f is well defined</u>:

Let, $x+k,\ y+k \in M/K$

$\Rightarrow x+k = y+k$

$\Rightarrow x-y \in k$

$\Rightarrow \Phi(x-y) \cancel{\in k} = 0$

$\Rightarrow \Phi(x) - \Phi(y) = 0$

$\Rightarrow \Phi(x) = \Phi(y)$

$\Rightarrow f(x+k) = f(y+k)$.

$\therefore$ f is well defined

(ii) <u>f is modul. homomorphism</u>:-

Let, $x+k,\ y+k \in M/K$ then, $r \in R$

Q. No.

$$f[(x+k)+(y+k)] = f[(x+y)+k]$$
$$= \Phi(x+y)$$
$$= \Phi(x) + \Phi(y)$$
$$= f(x+k) + f(y+k)$$

$\therefore$ f is well defined.

(iii) f is onto :-

Let, $\Phi(x) \in \mathrm{Img}\,\Phi$

then,

$\Phi(x) = f(x+k)$ for $x+k \in M/k$

$\therefore$ f is onto.

Hence, from, (i), (ii) & (iii) we observe that f is ~~onto~~ ~~homomorphism~~ R-module homomorphism.

$$\Rightarrow \frac{M}{\ker\Phi} \cong \mathrm{Img}\,\Phi.$$

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Q. No. 7.

Q.7 (b) Proof :- As A and B are submodules of R-module M. then,

$A \cap B$ is also a submodule of M

because, intersection of 2 submodules is again a submodule.

Therefore, $\frac{A}{A \cap B}$ is defined

as, B is also a submodule of A+B.

$f : A \longrightarrow \frac{A+B}{B}$ is defined by.

$f(a) = f(a+B)$

(i) f is well defined :-

Let, $a, b \in A$ then,

$\Rightarrow a = b$

$\Rightarrow a+B = b+B$

$\Rightarrow f(a) = f(b)$

$\therefore$ f is well defined :-

(ii) f is Homomorphism :-

Let, $a, b \in H$

$f(a+b) = f(a+b)$

$= f(a) + f(b)$

$= (a+B) + (b+B)$

$= f(a) + f(b)$

f is homomorphism

05

Q. No.				TOTAL				TOTAL
Marks								

Q. No.

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Q. No.	2 a)	b)	c)	TOTAL				TOTAL
Marks	04	04	01	= 09				

Q. No. 2

Q.2. a) Proof:- To show the $\mathbb{Z}$ has no composition series.

Let us consider a series,

$\{e\} = H_0 < H_2 < H_2 < \_\_\_ < H_n = G$ is the series for $\mathbb{Z}$

$= \mathbb{Z}$

then, $\frac{H_1}{H_0} = n\mathbb{Z}$

As, H_1 has n no. of element in $\mathbb{Z}$.

therefore, $n\mathbb{Z}$ cannot be a composition of $\mathbb{Z}$.

This shows that,

$\mathbb{Z}$ has no composition series.

04

Q.2) b) Sol^n:- $G = O(45) = 45$.

To show that 45 is not simple.

$45 = 3^2 \times 5$

Hence, by sylow's third theorem we observe that,

45 has 3-SSG of order 9

$\Rightarrow r / O(G)$ & $r \equiv 1 \pmod{P}$,

$\Rightarrow 3 / 45$ & $1\cancel{8} \equiv 1 \pmod{3}$

$\therefore$ 3-SSG Have a 1 element

consider,

H be a subgroup of G then,

i.e. It has a proper normal subgrp in G

$\therefore$ 45 is not simple.

04

Q. No.				TOTAL				TOTAL
Marks								

Q. No. 2.

2. (c). Sol^n:- $f(x) = 4 + 2x^2$

$f(x) \; 2(2+x^2)$ therefore, $4+2x^2$ is irreducible over Q.
but, a not irreducible over Z as.
2 is not a unit in Z.

0

Q. No.				TOTAL				TOTAL
Marks								

Q. No.

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